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The Dynamic Relationship of the GDP *Per capita* Among the Three Baltic States (1990-2021)

Agustín Alonso-Rodríguez

Department of Econometrics, Real Centro Universitario "Escorial-María Cristina", San Lorenzo de El Escorial (Madrid), Spain

Email address:
aalonso@rcumariacristina.com

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Abstract: The geographical situation in Europe of Estonia, Latvia and Lithuania, the Three Baltic States, forms an optimal environment for the study of the economic relationships present among them. The global magnitudes are very similar for the three States, with a little difference in favor of Lithuania regarding population and extension. The three States joined the European Union at the same time, May 1, 2004. A vector autoregressive model, a VAR model, relating the three economies in their temporal evolution is an appropriate model for this study. With the intervention of temporal lags, it is possible to formulate the dynamical relationship present in these economies regarding the percentage growth change in the respective gdp per capita. Our attention is directed to the evolution of this percentage growth rate for the period 1990-2020. The estimated VAR(2) model shows that the percentage change in the gdp per capita of Lithuania is dynamically related to the lagged growth changes of Estonia and Latvia in a direct way, with more complex dynamic relationships regarding the other two States, as explained in the Conclusion. This study is supplemented with the Impulse Response Analysis and the Forecast Error Variance Decomposition to measure the effects of random impulses in the evolution of the percentage growth change in the estimated model.

Keywords: Baltic States, GDP Per Capita, Percentage Change, VAR Models, Impulse Response Analysis, Forecast Error Variance Decomposition, Vars Statistical Package

1. Introduction

The geographic situation of the three Baltic States offers an attractive field of attention to study the economic ties among them. [4]

In this paper a model for the dynamic relationship among the *GDP per capita* in the three Baltic States: Estonia, Latvia and Lithuania, is presented. It is a vector autoregressive model for the three time series of the respective economies.

In the real economy, the relationship among variables is a reality. The analysis of the time series representing the different variables, allow us to better interpret the situation, as well as to get better predictions, better forecasts.

Let

$$z_t = (z_{1t}, z_{2t}, \dots, z_{kt})'$$

be a k-dimensional vector of *time series*, observed at equally spaced time points.

In this paper, let be

z_{1t}: GDP *per capita* of Estonia,

$z_{2,t}$: GDP *per capita* of Latvia, and

z_{3t}: GDP *per capita* of Lithuania.

The data are taken from the database *datosmacro* of the newspaper *Expansión*, from 1990 to 2021, annual data.

It has to be pointed out the relevance of the order of variables in a *VAR* model. Here, the variables are considered in alphabetical order.

The three time series are represented in figure 1.

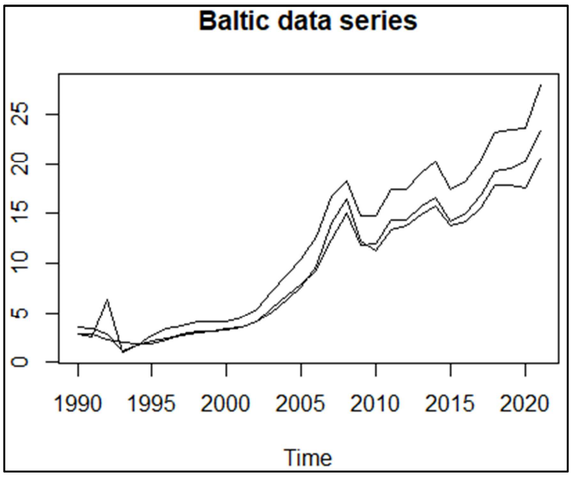

Figure 1. *Estonia, Latvia and Lithuania data series.*

For easy of interpretation, and to induce stationarity, let us transform the series into percentage growth changes. Figure 2.

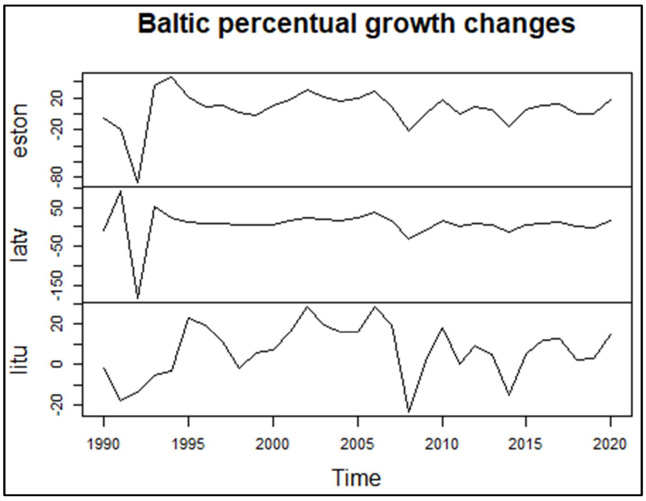

Figure 2. *Percentage growth changes of the Baltic data series.*

2. VAR Models

A VAR model [1, 2, 8, 11, 12, 13] for the multivariate vector of time series z_t of order p can be written as

$$z_t = \phi_0 + \Sigma_{i=1}^{p} \phi_i z_{t-i} + a_t \quad (1)$$

where ϕ_0 is a k-dimensional constant vector, ϕ_i are $k \times k$ matrices for $i > 0$, $\phi_p \neq 0$ and a_t a sequence of independent and identically distributed random vectors, with *mean = 0*, and matrix-covariance matrix Σ_a, positive-definite. [6, 14, 15].

Coming to our case, we can display (1) as a VAR(1)

$$\begin{pmatrix} z_{1,t} \\ z_{2,t} \\ z_{3,t} \end{pmatrix} = \begin{pmatrix} \phi_{10} \\ \phi_{20} \\ \phi_{30} \end{pmatrix} + \begin{pmatrix} \phi_{1,11} & \phi_{1,12} & \phi_{1,13} \\ \phi_{1,21} & \phi_{1,22} & \phi_{1,23} \\ \phi_{1,31} & \phi_{1,32} & \phi_{1,33} \end{pmatrix} \begin{pmatrix} z_{1,t-1} \\ z_{2,t-1} \\ z_{3,t-1} \end{pmatrix} + \begin{pmatrix} a_{1,t} \\ a_{2,t} \\ a_{3,t} \end{pmatrix} \quad (2)$$

2.1. Model Estimation

With the package *vars* of professor *Bernhard Pfaff* [9, 10] we proceed.

After a few intents, a VAR (2) model is estimated

```
#
# VAR Estimation Results:
# =========================
# Endogenous variables: eston, latv, litu
# Deterministic variables: const
# Sample size: 29
# Log Likelihood: -290.396
# Roots of the characteristic polynomial:
# 0.6956 0.6956 0.6134 0.6134 0.3322 0.3322
# Call:
# VAR(y = zt, p = 2)
#
#
# Estimation results for equation eston:
# ======================================
# eston = eston.l1 + latv.l1 + litu.l1 + eston.l2 + latv.l2 + litu.l2 + const
#
#         Estimate Std. Error t value Pr(>|t|)
# eston.l1  1.11366   0.20092   5.543 1.43e-05 ***
# latv.l1  -0.77653   0.09633  -8.061 5.21e-08 ***
# litu.l1   0.08559   0.22753   0.376   0.7104
# eston.l2 -0.16540   0.28199  -0.587   0.5635
# latv.l2  -0.16019   0.16302  -0.983   0.3365
# litu.l2   0.13180   0.20994   0.628   0.5366
# const     6.27888   2.46160   2.551   0.0182 *
# ---
# Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
#
#
# Residual standard error: 11.1 on 22 degrees of freedom
# Multiple R-Squared: 0.8168,     Adjusted R-squared: 0.7668
# F-statistic: 16.35 on 6 and 22 DF,  p-value: 4.219e-07
#
#
# Estimation results for equation latv:
# =====================================
# latv = eston.l1 + latv.l1 + litu.l1 + eston.l2 + latv.l2 + litu.l2 + const
#
#          Estimate Std. Error t value Pr(>|t|)
# eston.l1  1.697507  0.281997   6.020 4.66e-06 ***
# latv.l1  -1.401335  0.135209 -10.364 6.27e-10 ***
# litu.l1   0.435108  0.319342   1.363    0.187
# eston.l2  0.008264  0.395778   0.021    0.984
# latv.l2  -0.315104  0.228805  -1.377    0.182
# litu.l2   0.434763  0.294659   1.475    0.154
# const    -1.799923  3.454951  -0.521    0.608
# ---
# Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
#
```

```
#
# Residual standard error: 15.58 on 22 degrees of freedom
# Multiple R-Squared: 0.8734, Adjusted R-squared: 0.8389
# F-statistic: 25.29 on 6 and 22 DF, p-value: 8.168e-09
#
#
# Estimation results for equation litu:
# ======================================
# litu = eston.l1 + latv.l1 + litu.l1 + eston.l2 + latv.l2 + litu.l2 + const
#
#          Estimate Std. Error t value Pr(>|t|)
# eston.l1  0.38320    0.20371   1.881   0.0733.
# latv.l1  -0.11189    0.09767  -1.146   0.2643
# litu.l1   0.06319    0.23069   0.274   0.7867
# eston.l2  0.15863    0.28590   0.555   0.5846
# latv.l2   0.01493    0.16529   0.090   0.9289
# litu.l2  -0.31286    0.21286  -1.470   0.1558
# const     6.69871    2.49581   2.684   0.0136 *
# ---
# Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
#
#
# Residual standard error: 11.25 on 22 degrees of freedom
# Multiple R-Squared: 0.3794, Adjusted R-squared: 0.2101
# F-statistic: 2.241 on 6 and 22 DF, p-value: 0.07721
#
#
#
# Covariance matrix of residuals:
#       eston  latv  litu
# eston 123.2 140.6 112.8
# latv  140.6 242.7 136.0
# litu  112.8 136.0 126.6
#
# Correlation matrix of residuals:
#        eston   latv   litu
# eston 1.0000 0.8130 0.9035
# latv  0.8130 1.0000 0.7758
# litu  0.9035 0.7758 1.0000
```

2.2. Model Simplification

The estimated model *VAR(2)* presents parameters *no-statistically* significant for the usual significant level $\alpha = 0.05$. The simplification of a model is of interest when there is no prior knowledge to support those parameters. However, there exists no optimal method to simplify the fitted model. Cf. Tsay [15], p. 72.

Coming to our case,

```
#
# VAR Estimation Results:
# =========================
# Endogenous variables: eston, latv, litu
# Deterministic variables: const
# Sample size: 29
# Log Likelihood: -305.548
# Roots of the characteristic polynomial:
# 0.6133 0.6133 0.5747 0.4784 0.4784     0
# Call:
# VAR(y = zt, p = 2)
#
#
# Estimation results for equation eston:
# =========================================
# eston = eston.l1 + latv.l1 + latv.l2 + const
#
#          Estimate Std. Error t value Pr(>|t|)
# eston.l1  1.11410    0.13311   8.370 1.02e-08 ***
# latv.l1  -0.79430    0.07859 -10.106 2.59e-10 ***
# latv.l2  -0.22307    0.05402  -4.129 0.000355 ***
# const     7.18903    2.11937   3.392 0.002312 **
# ---
# Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
#
#
# Residual standard error: 10.6 on 25 degrees of freedom
# Multiple R-Squared: 0.8308, Adjusted R-squared: 0.8038
# F-statistic:  30.7 on 4 and 25 DF, p-value: 2.573e-09
#
#
# Estimation results for equation latv:
# ========================================
# latv = eston.l1 + latv.l1 + latv.l2 + litu.l2
#
#          Estimate Std. Error t value Pr(>|t|)
# eston.l1  1.90746    0.18870  10.108 2.58e-10 ***
# latv.l1  -1.43940    0.11753 -12.247 4.63e-12 ***
# latv.l2  -0.27799    0.08953  -3.105  0.00468 **
# litu.l2   0.50005    0.24010   2.083  0.04767 *
# ---
# Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
#
#
# Residual standard error: 15.23 on 25 degrees of freedom
# Multiple R-Squared: 0.8639, Adjusted R-squared: 0.8422
# F-statistic: 39.68 on 4 and 25 DF, p-value: 1.753e-10
#
#
# Estimation results for equation litu:
# ======================================
# litu = eston.l1 + latv.l1 + const
#
#          Estimate Std. Error t value Pr(>|t|)
# eston.l1  0.44358    0.13921   3.186  0.00373 **
# latv.l1  -0.15805    0.07769  -2.034  0.05223.
# const     6.07642    2.15013   2.826  0.00894 **
# ---
# Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
#
#
```

Residual standard error: 11.08 on 26 degrees of freedom
Multiple R-Squared: 0.4958, Adjusted R-squared: 0.4376
F-statistic: 8.521 on 3 and 26 DF, p-value: 0.0004155
#
#
#
Covariance matrix of residuals:
eston latv litu
eston 127.6 145.1 107.6
latv 145.1 263.3 138.4
litu 107.6 138.4 145.1
#
Correlation matrix of residuals:
eston latv litu
eston 1.0000 0.7918 0.7907
latv 0.7918 1.0000 0.7080
litu 0.7907 0.7080 1.0000

2.3. Model Checking

Before any considerations, let us perform the checking of model *m2*.

Regarding the residuals, we have figure 3. The residuals, to save space, are depicted jointly in figure 3.

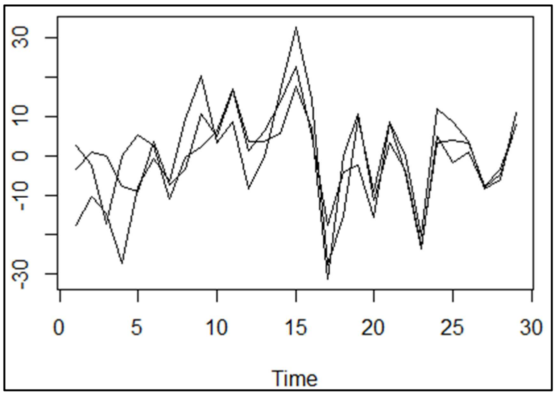

Figure 3. *Residuals for model m2.*

These residuals do not show a failure to comply with the fundamental suppositions.

Next, regarding the autocorrelations, they are depicted in figure 4.

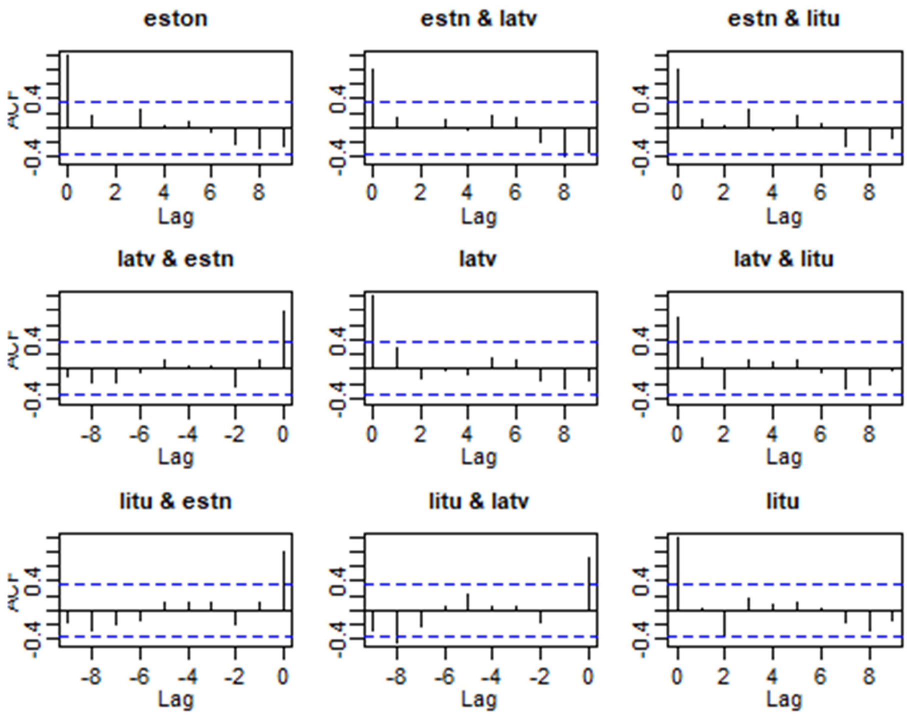

Figure 4. *Autocorrelations of residuals for model m2.*

The residuals, validate the estimated model. Other tests, as well, validate this model *m2*.

3. Interpretation

For the easy of interpretation, let us write separately the three estimated models.

For *Estonia*:

$$\hat{z}_{1t} = 7.189 + 1.114z_{1,t-1} - 0.794z_{2,t-1} - 0.223z_{2,t-2}$$

For *Latvia*:

$$\hat{z}_{2t} = 1.907z_{1,t-1} - 1.439z_{2,t-1} - 0.278z_{2,t-2} + 0.5z_{3,t-2}$$

For *Lithuania*:

$$\hat{z}_{3t} = 6.076 + 0.444z_{1,t-1} - 0.158z_{2,t-1}$$

In these models, all the estimated parameters are statistically significant.

Regarding *Estonia*, the percentage growth rate of the GDP *per capita* it is significantly related to its own past value, and to the past values of *latvia*, but it does not depend on the lagged percentage growth rate of the GDP *per capita* of Lithuania.

On the other hand, the percentage growth rate of the GDP *per capita* of *Latvia* is dynamically related to the percentage growth rate of its own past values and to the past values of the GDP *per capita* of *Estonia* and *Lithuania*.

Similarly, the percentage growth rate of the GDP *per capita* of *Lithuania* is dynamically related to the past values of the percentage growth rate of the GDP *per capita* of *Estonia* and *Latvia*.

All three percentage growth rate are directly dynamically correlated, as shown by the above estimated correlation matrix $\hat{\Sigma}_a$ of residuals.

Validated the model, let us use it for prediction.

4. Prediction

Let us consider the horizon of 5 years. These forecasts are depicted in figure 5.

Let us consider the horizon of 5 years. These forecasts are depicted in figure 4. Delete

```
# $eston
#        fcst     lower    upper      CI
# [1,] 14.319727  -6.44634 35.08579 20.76607
# [2,]  9.832307 -15.87282 35.53743 25.70512
# [3,]  5.505007 -20.66223 31.67224 26.16723
# [4,]  8.142698 -19.87554 36.16094 28.01824
# [5,]  9.298608 -18.80202 37.39924 28.10063
#
# $latv
#        fcst     lower    upper      CI
# [1,] 12.447382 -17.41144 42.30621 29.85883
# [2,] 12.415286 -27.75897 52.58954 40.17426
# [3,]  3.034012 -38.24664 44.31467 41.28065
# [4,]  7.913081 -35.97271 51.79888 43.88579
# [5,]  7.536553 -36.49083 51.56394 44.02738
#
# $litu
#        fcst     lower    upper      CI
# [1,] 11.219200 -10.49856 32.93696 21.71776
# [2,] 10.461057 -12.12208 33.04420 22.58314
# [3,]  8.475587 -14.24674 31.19792 22.72233
# [4,]  8.038808 -14.85744 30.93505 22.89624
# [5,]  8.437691 -14.57770 31.45309 23.01539
```

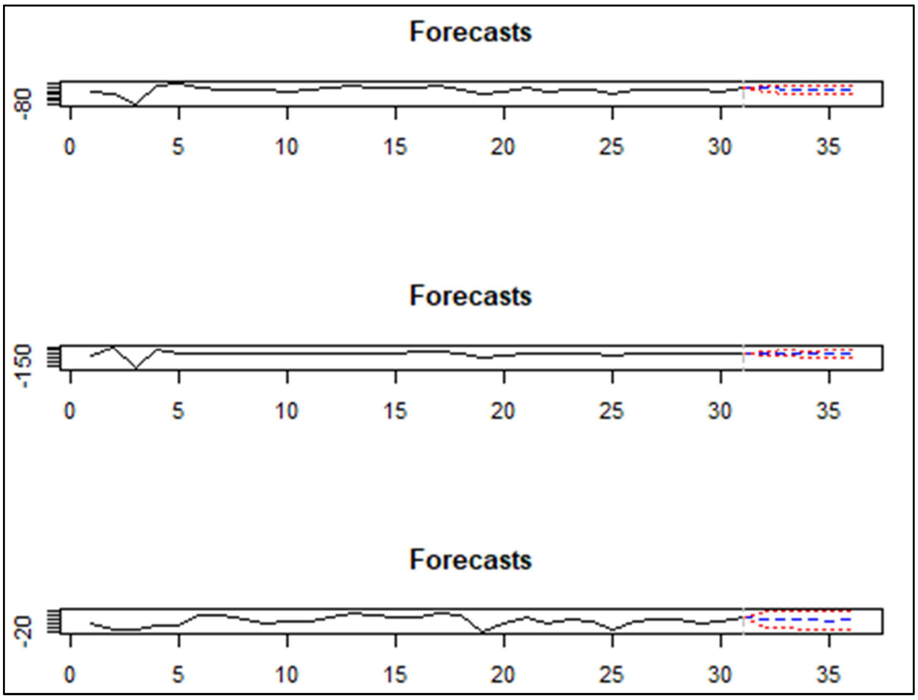

Figure 5. *Forecasts of the percentage growth changes in GDP per capita; Estonia, Latvia, Lithuania: 2022-2026.*

5. Impulse Response Analysis

Since all variables in a VAR model depend on each other, the estimated coefficients provide limited information on the reaction of the system to a shock. In order to get a better picture of the model's dynamic behavior, impulse responses are used. The *impulse response functions* [3, 7] show the effects of shocks on the adjustment path of the variables. Using the vars package, Paff [10], we get for model *m2.*

```
#
#
# Impulse response coefficients
# $eston
```

```
#            latv        litu
# [1,] 12.848305830  9.52482775
# [2,]  3.049869846  2.97931002
# [3,]  1.336464218  0.57265939
# [4,] -6.316836926 -1.38214416
# [5,]  0.075099473 -1.07879754
# [6,]  0.007450426 -0.23264402
# [7,]  0.945338245  0.35146027
# [8,]  0.167038613  0.23340390
# [9,]  0.071046614  0.06627865
#
#
# Lower Band, CI= 0.95
# $eston
#          latv       litu
# [1,]   7.275514  4.0616149
# [2,]  -5.123702 -0.2905359
# [3,]  -3.801577 -3.7056445
# [4,] -11.118909 -3.2470032
# [5,]  -2.735582 -2.9096469
# [6,]  -1.897026 -1.5308566
# [7,]  -1.147467 -0.4776698
# [8,]  -2.070017 -0.5910830
# [9,]  -1.341507 -0.6709194
#
#
# Upper Band, CI= 0.95
# $eston
#         latv       litu
# [1,] 15.656911 12.9353556
# [2,]  8.938834  5.3797022
# [3,]  7.206170  2.8760377
# [4,] -2.080251  0.3725622
# [5,]  3.566242  0.8766221
# [6,]  2.834761  1.1136835
# [7,]  3.224141  1.7506425
# [8,]  1.461116  1.0302961
# [9,]  1.344411  0.6200428
```

Result depicted in figure 6.

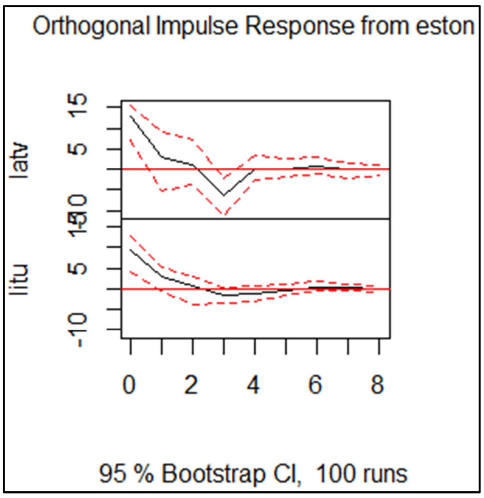

Figure 6. *Impulse response in the system of a shock in Estonia.*

The advantage of considering the impulse response functions, and not just VAR coefficients, is that they show the size of the impact of one standard deviation shock in the variables plus the rate at which the shock disipates, allowing for interdependencies. In figure 6, the impact is positive, and disipates rapidly.

The interpretation of figure 6 is straightforward. An impulse (shock) to *latv* and to *litu* at time zero, has large effects the next period, but the effects become smaller and smaller as time passes. The dotted lines show the 95% interval estimates of these effects.

6. Forecast Error Variance Decomposition

Another way of studying the effects of shocks in the variables, is to consider the contributions of a shock in the forecast error variance. This decomposition is often expressed in proportional terms. [5, 6, 15].

Forecast error variance decomposition estimates the contribution of a shock in each variable to the response in all variables.

Coming to our case, Paff [10], we have

```
# $eston
#        eston      latv       litu
# [1,] 1.1363636 0.0000000 0.00000000
# [2,] 0.7744959 0.3618677 0.00000000
# [3,] 0.7864738 0.3498898 0.00000000
# [4,] 0.7932950 0.3078717 0.03997962
# [5,] 0.7898553 0.3114319 0.03982410
# [6,] 0.7899782 0.3103747 0.04088852
# [7,] 0.7906834 0.3092434 0.04140025
# [8,] 0.7904358 0.3095119 0.04137695
#
# $latv
#        eston      latv       litu
# [1,] 0.7112831 0.4250806 0.00000000
# [2,] 0.4150485 0.7213152 0.00000000
# [3,] 0.3971249 0.7119717 0.02919138
# [4,] 0.4309642 0.6332548 0.07934215
# [5,] 0.4282079 0.6345924 0.08082711
# [6,] 0.4281700 0.6346330 0.08082413
# [7,] 0.4289791 0.6335332 0.08117163
# [8,] 0.4288413 0.6336924 0.08114667
#
# $litu
#        eston       latv       litu
# [1,] 0.7388913 0.02113659 0.4217903
# [2,] 0.7502066 0.03811093 0.3900837
# [3,] 0.7434837 0.04908481 0.3853193
# [4,] 0.7462302 0.04956999 0.3818583
# [5,] 0.7469637 0.05111030 0.3793784
# [6,] 0.7468555 0.05137984 0.3791999
# [7,] 0.7468952 0.05139248 0.3791462
# [8,] 0.7469752 0.05141411 0.3790359
```

In our case, the forecast error variance decomposition is depicted in figure 7.

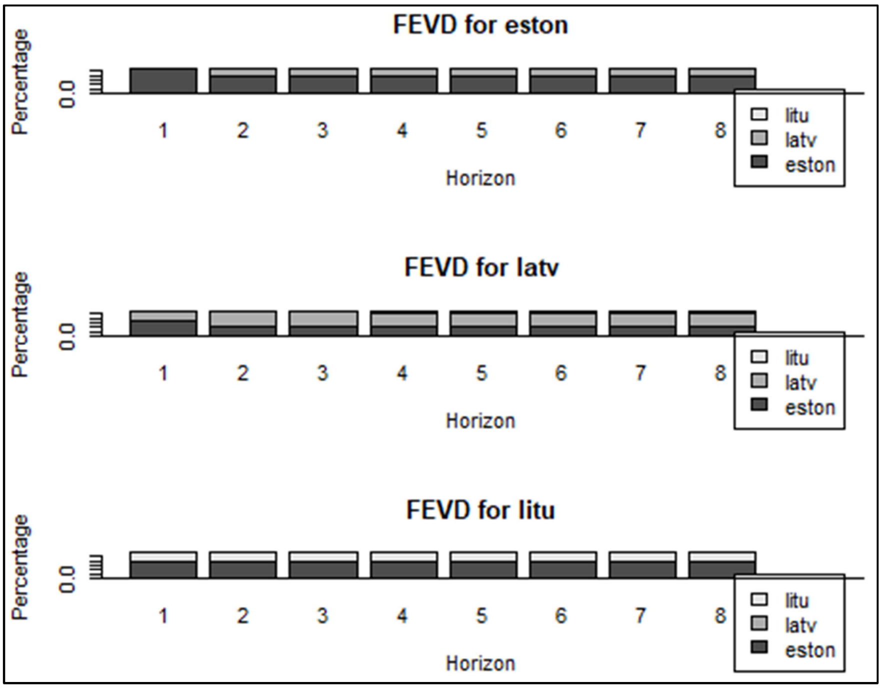

Figure 7. *Variance error decomposition for model m2.*

Figure 7 shows that almost 100% of the variance in *eston* is caused by *eston* itself, while around 60% in the variance of *latv* is caused by *latv* itself and similar can be said for *litu*.

7. Conclusion

The estimated VAR(2) model shows the existence of an overall dynamic relationship in the percentage change of the gdp per capita in the Three Baltic States. But at the same time, and during the period 1990-2020, the model reveals the following notes of differentiation: Estonia is not dynamically related to Lithuania, Latvia is dynamically related to Estonia and Lithuania and Lithuania is dynamically related to Estonia and Latvia. With independency of these distinctions, the estimated correlation matrix of the error term of the model shows a strong dynamic relationship among all of them.

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